3D-Pictogram of Risk Measures in Epidemiology

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Objective

To visualize health impacts of sets of exposures with respect to one or more outcomes by simultaneously graphing relative risk, attributable risk, and exposure prevalence along with multidimensional confidence volumes.

Summary/Abstract

Background
Population health impact of an exposure can be described as the absolute or relative number of individuals who develop a certain disease due to the exposure. Several measures are combined to obtain these estimates: the absolute risk (in the non-exposed or in the total population), the relative risk, the population size, and the exposure prevalence. Previous attempts to visualize population health impacts have not considered all these dimensions simultaneously.

Epidemiological risk measures
If the exposure prevalence p_1 is known then the number of cases attributable to the exposure is determined using relative risk RR and absolute risk of the total population R_p or of non-exposed group R_0.
This enables a comprehensive assessment of the health impact of an exposure. It is crucial that the calculation of the attributable risk is conducted within specific age and sex groups whenever exposure prevalence and absolute and relative risks are not uniformly distributed among strata.

Graphical method
A 3-dimensional plot enables the visualization of the attributable cases in the exposed population, together with the relative risk and the exposure prevalence.
If the precision of the risks and exposure prevalence is available, the 3D-scatter plot can be extended to the 3D-ellipsoids, representing confidence volumes of the attributable cases according to the other two dimensions which are visualized.

Computational implementation
Excel is used to capture and pre-process the data and the R-rgl package to generate the 3D-pictogram.

Definition and Notation of Risk Measures

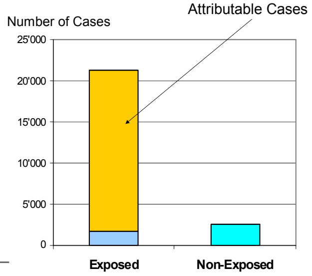
- Population
  - N_p: population size
  - D_p: number of cases in the total population
  - p_1: exposure prevalence in the total population
- Absolute risks
  - R_p: disease rate in population
  - R_0: disease rate in non-exposed
  - R_1: disease rate in exposed
- Relative risk
  - $RR = R_1/R_0$ strength of the exposure-outcome association
- Attributable cases due to the exposure
  - Absolute $AC = D_p - N_p * R_0$
  - Relative $PAP = AC / D_p$ (= population attributable risk proportion)
- Key formulas
  - $R_0 = R_p / [\, p_1 * (RR - 1) + 1 \,]$
  - $R_1 = RR * R_0$

Calculation of Risk Estimates

Population size = 50'000'000
Exposure prevalence p_1 = 30%

R_p = 47.7 per 100'000 individuals
Relative risk RR = 12.70

	Cases Due to Exposure	Cases Due to Others Causes	Total Cases	Total Population	Pct. (%)
Exposed	**19'651** (4)	**1'680** (3)	**21'331** (3)+(4)	20'000'000	40
Non-Exposed		**2'519** (2)	**2'519**	30'000'000	60
Total		**4'199** (2)+(3)	**23'850** (1)	50'000'000	100



R_p	R_0	R_1	RR	AC	PAP
47.70	8.40	106.65	12.70	19'651	82.4

Unit: R_0, R_p, R_1 are per 100'000 individuals and PAP is in %

Formulas

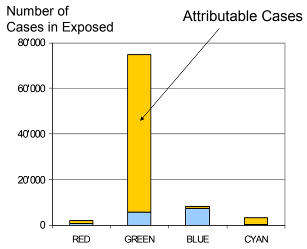
$D_p = N_p * R_p$ (1)
$D_{0\_other} = N_p * p_1 * R_0$ (3)
$D_0 = N_p * (1-p_1) * R_0$ (2)
$AC = D_p - N_p * R_0$ (4)

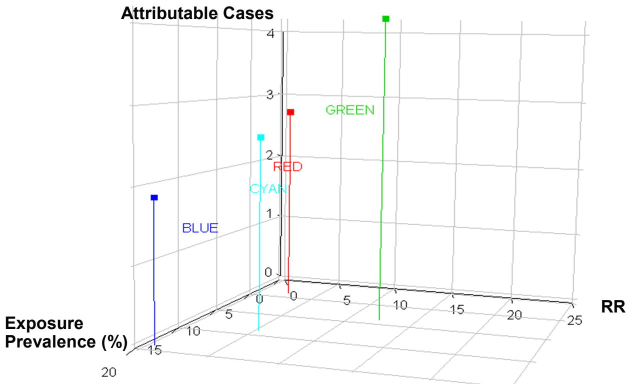
Example of 4 Arbitrary Exposure-Outcome Relationships

Population size = 50'000'000

	p_1	R_0	R_p	R_1	RR	AC	PAP
RED	5	29.9	32.9	90.9	3.04	1'525	9.3
GREEN	10	118	256	1499	12.7	69'030	53.9
BLUE	20	74.7	76.8	85.2	1.14	1'046	2.72
CYAN	15	7.80	13.5	45.8	5.87	2'849	42.2

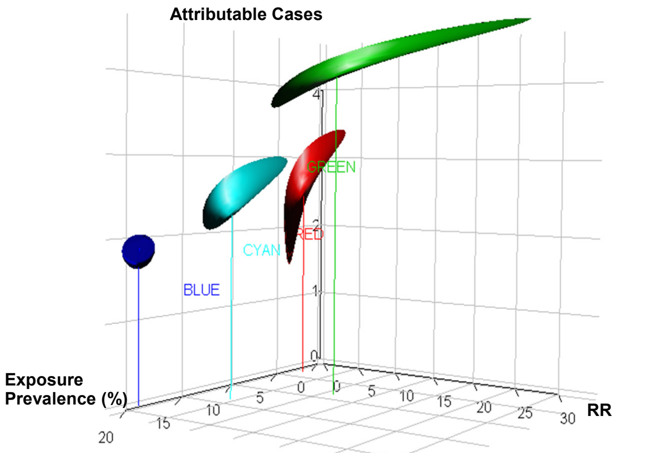
Unit: R_0, R_p, R_1 are per 100'000 individuals
p_1 and PAP are in %





3-D Pictogram

Assumptions: Ln(RR) and R_p are normally distributed and 95% confidence intervals of RR and R_p are available



Conclusions

The 3D-ellipsoid pictogram supports a comprehensive visual comparison of different exposure-outcome pairs in term of attributable cases as well as relative risks, and exposure prevalence.

3D-Ellipsoid Formulation

Assumptions

- RR is log-normal distributed: $\mathrm{Ln}(RR) \sim N(\mu_{RR}, \sigma^2_{RR})$
- R_p is normal distributed
- RR, R_p, and p_1 are mutually independent

Conclusions

- $\mathrm{std}(R_p) = [\text{Upper 95\% CI}(R_p) - R_p] / 1.96$
- $\mathrm{std}(\mathrm{Ln}(RR)) = \mathrm{Ln}[\text{Upper 95\% CI}(RR)/RR] / 1.96$

Definition of 3D-Ellipsoid: E(X,Y,Z)

E(X,Y,Z):

$$\chi_{1-\alpha} = (X-\mu_x)^2/s^2_x + (Y-\mu_y)^2/s^2_y + (Z-\mu_z)^2/s^2_z - 2\,\rho(X,Y)\,(X-\mu_x)\,(Y-\mu_y)/(s_X\, s_Y) - 2\,\rho(X,Z)\,(X-\mu_x)\,(Z-\mu_z)/(s_X\, s_Z) - 2\,\rho(Y,Z)\,(Y-\mu_y)\,(Z-\mu_z)/(s_Y\, s_Z)$$

where $\chi_{1-\alpha}$ is the $1-\alpha$ percentile of the χ distribution with 3 degrees of freedom and $\rho(X,Y)$ is the correlation between X and Y

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