Modeling heat and fluid flow in porous media

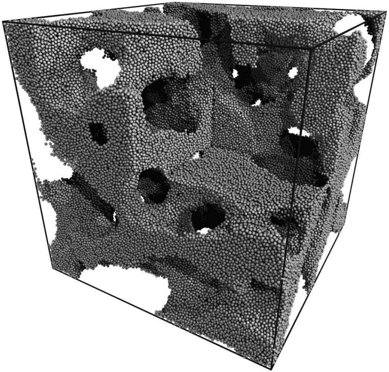
David J. Lopez Penha*, Lilya Ghazaryan*,
Bernard J. Geurts*, Steffen Stolz†,* & Markus Nordlund†

*Dept. of Applied Mathematics
University of Twente, Enschede
The Netherlands

†Philip Morris International R&D
Philip Morris Products S.A., Neuchâtel
Switzerland

ECCOMAS CFD 2010, Lisbon, Portugal
June 14–17, 2010

Porous media



Amorphous nano-porous material

Source: http://gubbins.ncsu.edu/research.html

- Porous media may have complicated pore geometries
- Difficult to build practicable body-fitted grids

Goals

1. Develop "gridding-free" method for computing heat & fluid flow in porous media
2. Method allows arbitrary pore geometries

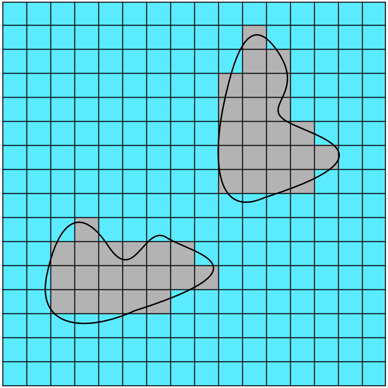
Goals

1. Develop "gridding-free" method for computing heat & fluid flow in porous media
2. Method allows arbitrary pore geometries

Outline

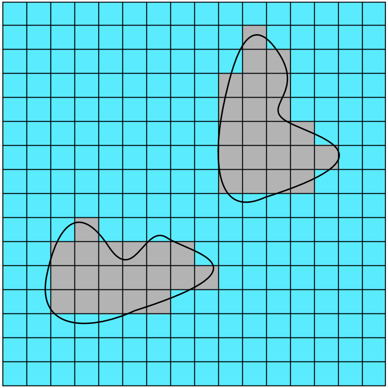
Outline

Representing complex geometries



- Cartesian grid representation of fluid & solid domains
- Trade-off: large spatial resolutions — efficient numerical algorithms

Representing complex geometries



- Cartesian grid representation of fluid & solid domains
- Trade-off: large spatial resolutions — efficient numerical algorithms

Fluid dynamics

- Incompressible Navier-Stokes equations for fluid & solid domains:

$$\nabla \cdot \mathbf{u} = 0, \qquad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + \mathbf{f}$$

- Methodology: immersed boundary method
- Force $\mathbf{f}$: approximates no-slip condition (volume-penalization)

$$\mathbf{f} = -\frac{1}{\epsilon}\Gamma(\mathbf{x}) \cdot \mathbf{u}, \qquad \epsilon \ll 1$$

- $\Gamma(\mathbf{x})$: phase-indicator function ($\Gamma = 1$ in solid; $\Gamma = 0$ in fluid)

Fluid dynamics

- Incompressible Navier-Stokes equations for fluid & solid domains:

$$\nabla \cdot \mathbf{u} = 0, \qquad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + \mathbf{f}$$

- Methodology: immersed boundary method
- Force $\mathbf{f}$: approximates no-slip condition (volume-penalization)

$$\mathbf{f} = -\frac{1}{\epsilon} \Gamma(\mathbf{x}) \cdot \mathbf{u}, \qquad \epsilon \ll 1$$

- $\Gamma(\mathbf{x})$: phase-indicator function ($\Gamma = 1$ in solid; $\Gamma = 0$ in fluid)

Discretization

- Symmetry-preserving finite-volume method
- Staggered grid (uniform Cartesian)
- Implicit time-integration of force $\mathbf{f}$

Outline

Conjugate heat transfer

- Single temperature equation for fluid & solid domains:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla \cdot (\alpha \nabla T)$$

- Thermal diffusivity α: discontinuous if $\alpha_f \neq \alpha_s$

$$\alpha(\mathbf{x}) = (1 - \Gamma)\alpha_f + \Gamma \alpha_s$$

- Solid domains: convective term vanishes $\Longrightarrow$ diffusion only

Conjugate heat transfer

- Single temperature equation for fluid & solid domains:

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla \cdot (\alpha \nabla T)$$

- Thermal diffusivity α: discontinuous if $\alpha_f \neq \alpha_s$

$$\alpha(\mathbf{x}) = (1 - \Gamma)\alpha_f + \Gamma \alpha_s$$

- Solid domains: convective term vanishes $\Longrightarrow$ diffusion only

Discretization

- Physics: heat flux $\alpha \nabla T$ continuous $\Longrightarrow$ ∇T discontinuous at jumps in α $\Longrightarrow$ special care discretizing ∇T on jump interfaces
- Auxiliary temperatures $\{T^x_{i,j}, T^y_{i,j}\}$ on cell surfaces:

$$T^x_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i+1,j} T_{i+1,j}}{\alpha_{i,j} + \alpha_{i+1,j}}$$

$$T^y_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i,j+1} T_{i,j+1}}{\alpha_{i,j} + \alpha_{i,j+1}}$$

Discretization

- Physics: heat flux $\alpha \nabla T$ continuous $\Longrightarrow$ ∇T discontinuous at jumps in α $\Longrightarrow$ special care discretizing ∇T on jump interfaces
- Auxiliary temperatures $\{T^x_{i,j}, T^y_{i,j}\}$ on cell surfaces:

$$
\begin{aligned}
T^x_{i,j} &= \frac{\alpha_{i,j} T_{i,j} + \alpha_{i+1,j} T_{i+1,j}}{\alpha_{i,j} + \alpha_{i+1,j}} \\
T^y_{i,j} &= \frac{\alpha_{i,j} T_{i,j} + \alpha_{i,j+1} T_{i,j+1}}{\alpha_{i,j} + \alpha_{i,j+1}}
\end{aligned}
$$

Discretization

- Physics: heat flux $\alpha \nabla T$ continuous $\Longrightarrow$ ∇T discontinuous at jumps in α $\Longrightarrow$ special care discretizing ∇T on jump interfaces
- Auxiliary temperatures $\{T^x_{i,j}, T^y_{i,j}\}$ on cell surfaces:

$$T^x_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i+1,j} T_{i+1,j}}{\alpha_{i,j} + \alpha_{i+1,j}}$$

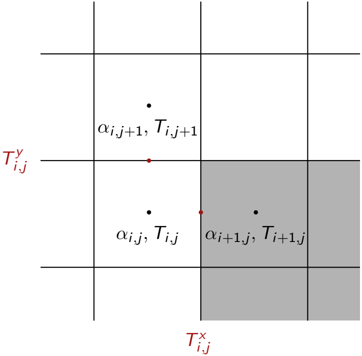
$$T^y_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i,j+1} T_{i,j+1}}{\alpha_{i,j} + \alpha_{i,j+1}}$$

Discretization

- Physics: heat flux $\alpha\nabla T$ continuous $\Longrightarrow$ ∇T discontinuous at jumps in α $\Longrightarrow$ special care discretizing ∇T on jump interfaces
- Auxiliary temperatures $\{T^x_{i,j}, T^y_{i,j}\}$ on cell surfaces:

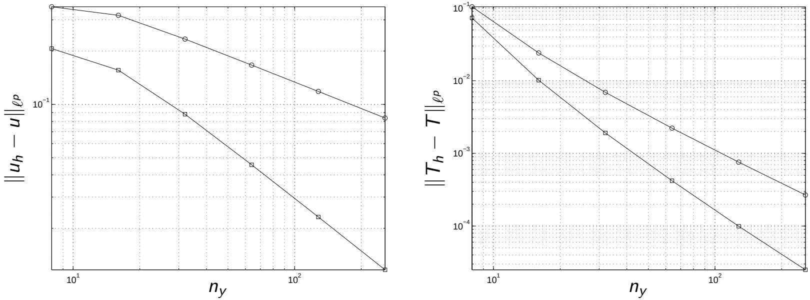
$$T^x_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i+1,j} T_{i+1,j}}{\alpha_{i,j} + \alpha_{i+1,j}}$$

$$T^y_{i,j} = \frac{\alpha_{i,j} T_{i,j} + \alpha_{i,j+1} T_{i,j+1}}{\alpha_{i,j} + \alpha_{i,j+1}}$$



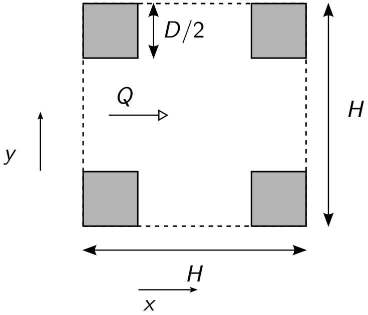
Outline

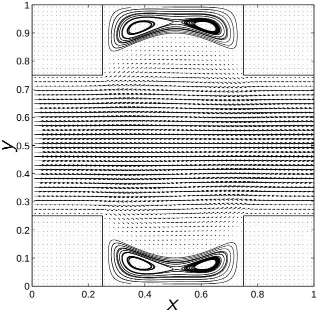
Plane-Poisuielle flow with isothermal walls



- ℓ^p-norm of error in velocity & temperature ($p = \{2, \infty\}$)
- Velocity: first-order in ℓ^∞
- Temperature: second-order in ℓ^∞

Porous medium: inline arrangement of squares





- Porosity: $\phi = 0.75 \implies 25\%$ solid grid cells
- Literature: Nakayama et al., J. Heat Transfer, 124, 746–753 (2002)

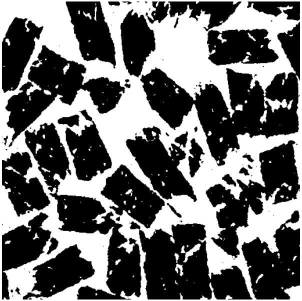
Porous medium: inline arrangement of squares

- Gradient average pressure $\partial\langle p\rangle^f/\partial x$ vs. grid resolution:

$(n_x \times n_y)\backslash$Re	10	100	600
32×32	6.65	0.711	0.124
64×64	7.16	0.768	0.135
128×128	7.45	0.800	0.143
181×181 (Nakayama et al.)	**7.82**	**0.835**	**0.154**

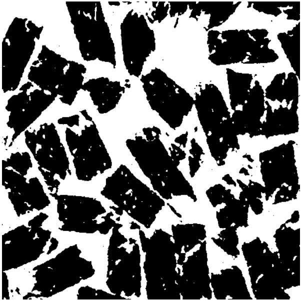
Outline

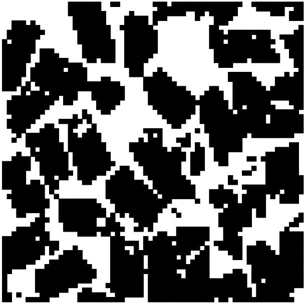
Realistic porous medium



- Left: solid data from μCT-imaging (black: solid material)
- Right: Cartesian grid representation using $\Gamma(\mathbf{x})$ [grid: $(32)^2$]

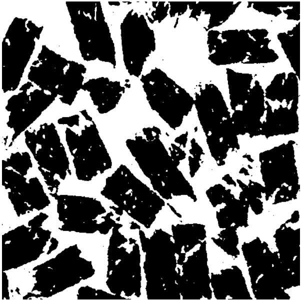
Realistic porous medium

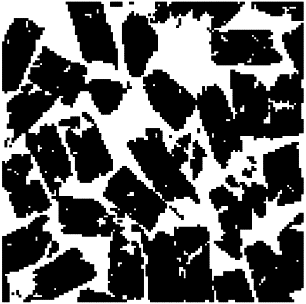




- Left: solid data from μCT-imaging (black: solid material)
- Right: Cartesian grid representation using $\Gamma(\mathbf{x})$ [grid: $(64)^2$]

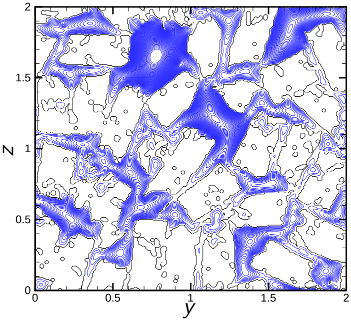
Realistic porous medium

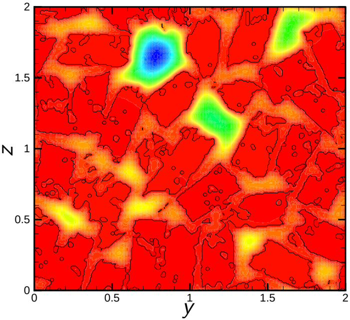




- Left: solid data from μCT-imaging (black: solid material)
- Right: Cartesian grid representation using $\Gamma(\mathbf{x})$ [grid: $(128)^2$]

Simulated field variables





- Left: contours out-of-plane velocity [grid: $(256)^2$]
- Right: fluid-solid temperature [grid: $(256)^2$]

Outline

Conclusions

Immersed boundary method:

- Reliably capture laminar fluid dynamics with adequate grid resolution
- Include arbitrarily shaped solid objects using Cartesian grid representation
- First-order accurate in fluid dynamics & second-order accurate in heat transfer