Modeling Effective Properties of Porous Media Using Tomographic Reconstruction

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Background

- Porous media are encountered in a wide variety of technologies, e.g., for filtration, heat transfer, and chemical reaction.
- Large-scale computations are generally performed using transport equations in terms of mean flow variables.
- These equations contain unknown *effective transport properties* of the porous medium which are vital to performing accurate computations.

Objectives

- Develop a computer model for simulating the detailed transport of momentum and energy in a representative sample of the porous medium [1].
- Compute the directional permeability and Nusselt number as functions of the system properties.

Modeling strategy

- Use periodic unit cell that is representative of the porous medium under consideration.
- Solve the transport equations using periodicity conditions on $\{\boldsymbol{u}, p, T_f, T_s\}$:

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{u} - \frac{1}{\epsilon} \Gamma \boldsymbol{u}, \qquad \epsilon \ll 1$$

$$\frac{\partial T_f}{\partial t} + \nabla \cdot (\boldsymbol{u} T_f) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \nabla^2 T_f, \qquad R_{c_p} \frac{\partial T_s}{\partial t} = \frac{R_\lambda}{\mathrm{Re}\,\mathrm{Pr}} \nabla^2 T_s + 1$$

with $\Gamma(\boldsymbol{x}) = 1$ for $\boldsymbol{x}$ in the solid and $\Gamma(\boldsymbol{x}) = 0$ for $\boldsymbol{x}$ in the fluid. For the material properties: $R_{c_p} \equiv c_{p,s}/c_{p,f}$ and $R_\lambda \equiv \lambda_s/\lambda_f$.

- Process the steady-state solution to obtain effective transport properties (direction $\boldsymbol{n}$):

$$\frac{1}{k_{\boldsymbol{n}}} = -\frac{\left(\boldsymbol{n} \cdot \nabla \langle p \rangle^f\right) \mathrm{Re}}{|\langle \boldsymbol{u} \rangle|}, \qquad \mathrm{Nu} = \frac{\int_{A_{sf}} R_\lambda \nabla T_s \cdot \mathrm{d}\boldsymbol{A}}{A_{sf}\left(\langle T_s \rangle^s - \langle T_f \rangle^f\right)}$$

Structured models of porous media

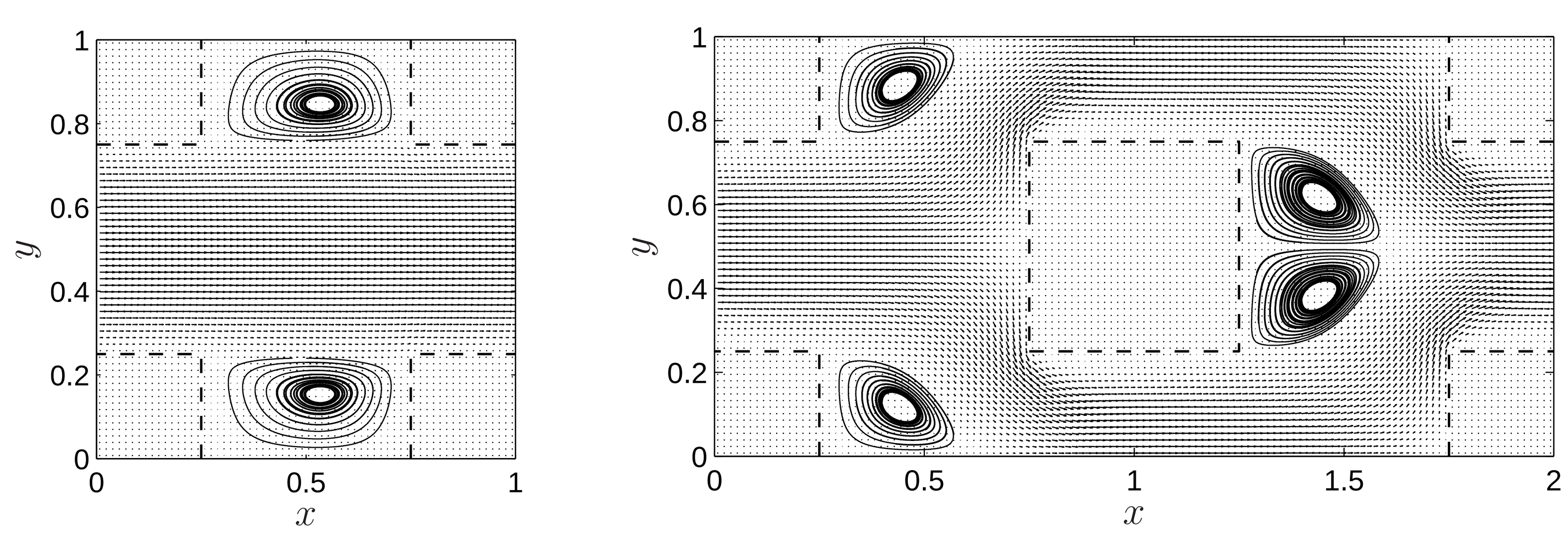

Fig. 1. Fully-developed velocity vectors for an inline and a staggered arrangement of square rods ($\mathrm{Re} = 100$; $\langle u \rangle = 1$).

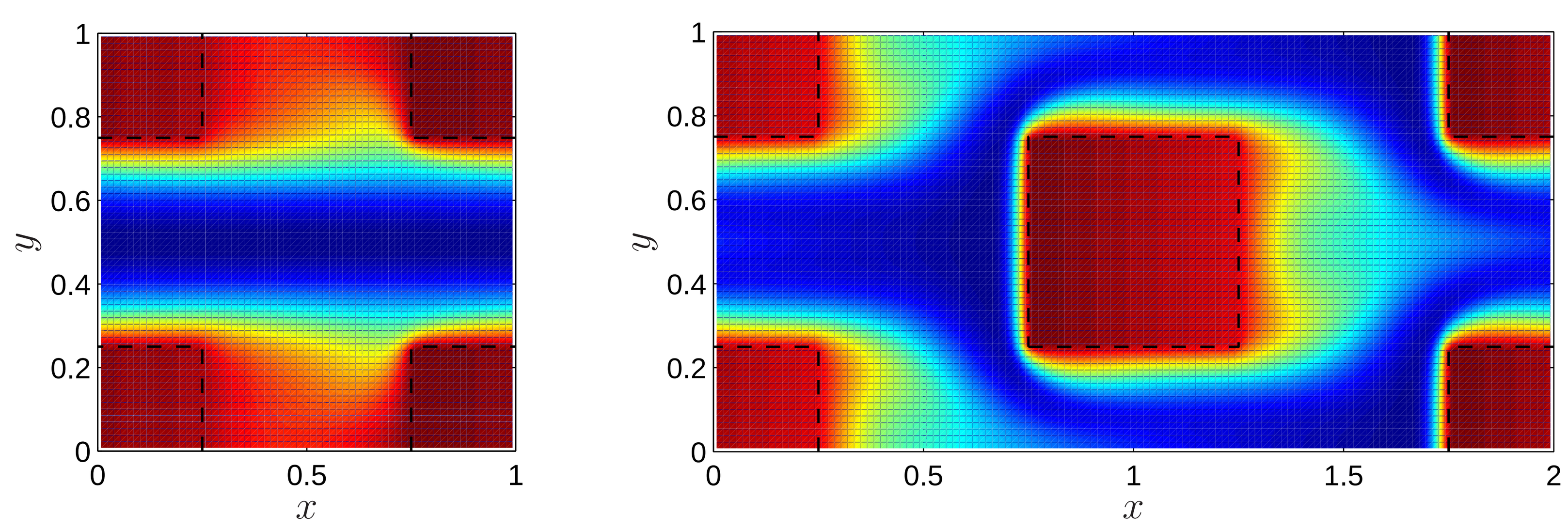

Fig. 2. Fields of fully-developed temperature fluctuation over a mean temperature field ($\mathrm{Re}\,\mathrm{Pr} = 100$; $R_{c_p} = R_\lambda \to \infty$).

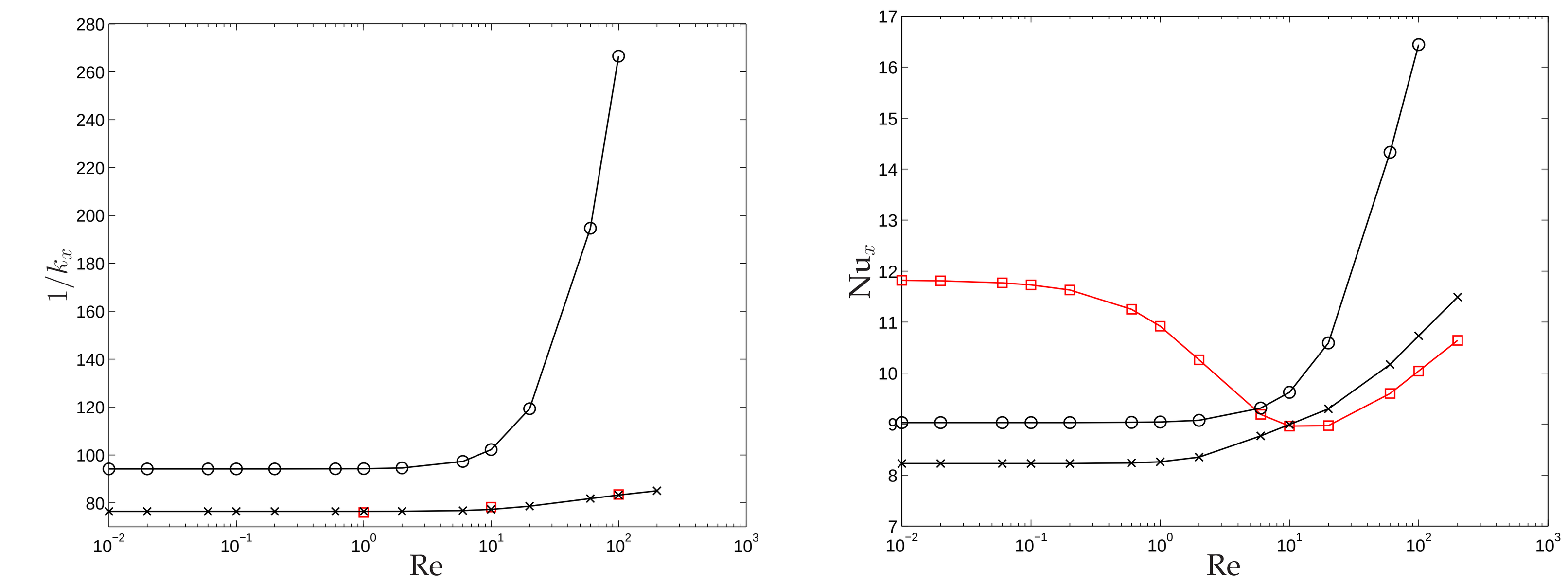

Fig. 3. Permeability (k_x) and Nusselt number (Nu_x) predictions along the x-axis versus the Reynolds number. (◯)-markers represent the staggered arrangement and ($\times$)-markers the inline arrangement ($\mathrm{Pr} = 1$; $R_{c_p} = R_\lambda \to \infty$). Red data points represent inline data from Ref. [2].

Realistic 3D porous medium

- Geometry extracted from μCT-imaging (computer tomography).
- Represented on a Cartesian grid using the Γ-function.

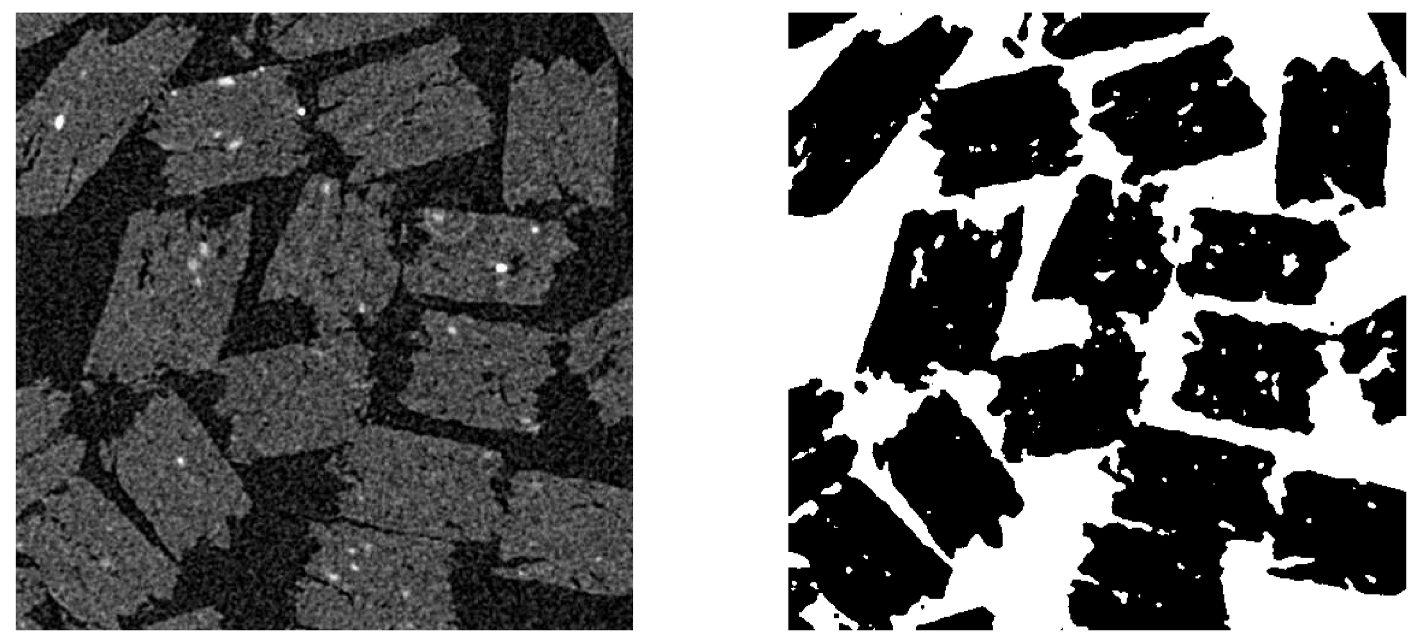

Fig. 4. Raw and post-processed (black indicates solid objects; white are interstitial channels) μCT-image of a single cross section of a porous medium.

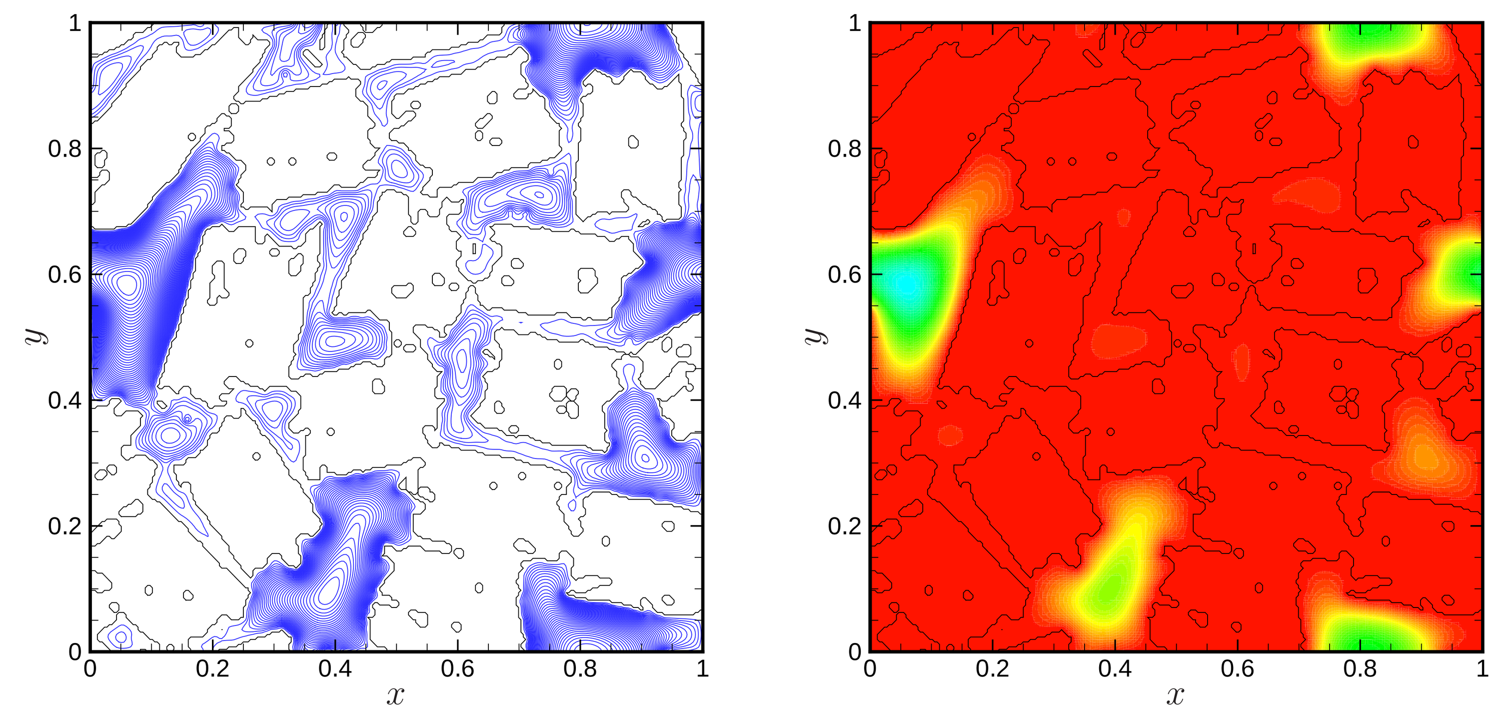

Fig. 5. Contour plot of the simulated out-of-plane velocity component and the solid–fluid temperature field.

Summary & outlook

- Developed computer model for momentum and energy transport.
- Improve accuracy of domain representation for application to realistic porous media.

References

[1] D. J. Lopez Penha, B. J. Geurts, S. Stolz and M. Nordlund, Computing the apparent permeability of an array of staggered square rods using volume-penalization, *Comput. Fluids*, (2011), `http://dx.doi.org/10.1016/j.compfluid.2011.08.011`

[2] A. Nakayama, F. Kuwahara, T. Umemoto and T. Hayashi, Heat and fluid flow within an anisotropic porous medium, *J. Heat Transfer*, **124**, 746–753 (2002).

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