Computing pore-scale fluid & heat transport in realistic porous media

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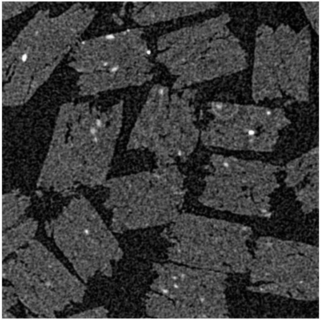
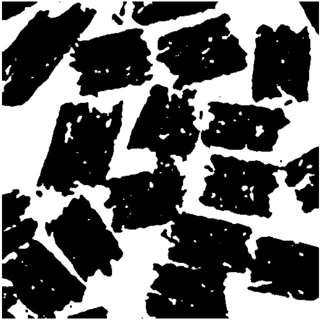
Example of realistic porous medium





- **Left:** Micro-computed-tomography (μCT) $\longrightarrow$ gray-scale images of cross-sections
- **Right:** Binary color scale & noise reduction (black $=$ solid matrix; white $=$ fluid channels)

Goals & approximation strategy

Goals:

- Reconstruct 3D porous media from 2D μCT-scans
- Simulate laminar fluid & energy transport (velocity, pressure & temperature)
- Compute permeability & heat transfer coefficients used in engineering models

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Numerical approximation strategy:

- Flexible to allow arbitrary pore geometry
- Require non-body conforming grid ("gridding-free" strategy)
- **Methodology:** Immersed boundary method

Part I: Numerical approximation strategy

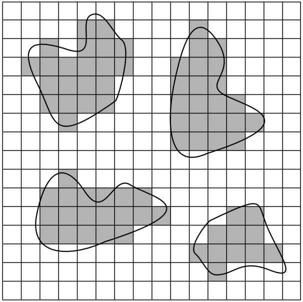
Part II: Simulation results for realistic porous medium

Part III: Conclusions & outlook

Part I: Pixelated structure of images

- **Close-up:** Image formed by individual square pixels
- Adopt meshing strategy that mimics pixelated structure

Part I: Cartesian grid representation



- Immerse solid domains into Cartesian grid
- Identify fluid/solid using **phase indicator function**:

$$\Gamma(\mathbf{x}) = \begin{cases} 0, & \text{for } \mathbf{x} \text{ in fluid domain (fluid fraction} \geq 50\%) \\ 1, & \text{for } \mathbf{x} \text{ in solid domain (solid fraction} > 50\%) \end{cases}$$

Part I: Transport equations

- **Transport equations:**

$$\begin{aligned}\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \mathbf{f} \\ \frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T &= \nabla\cdot(\alpha\nabla T)\end{aligned}$$

- $\mathbf{f}$: Force $\mathbf{u}\to 0$ in solid domains by volume penalization

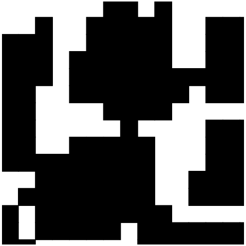
$$\mathbf{f} = -\frac{1}{\epsilon}\Gamma(\mathbf{x})\,\mathbf{u}, \qquad \epsilon \ll 1$$

- **Methodology:** Immersed boundary method
- **Thermal diffusivity** α**:** Discontinuous if $\alpha_f \neq \alpha_s$

$$\alpha(\mathbf{x}) = \left[1-\Gamma(\mathbf{x})\right]\alpha_f + \Gamma(\mathbf{x})\alpha_s$$

Part II: Geometrical convergence

(a) μCT-image

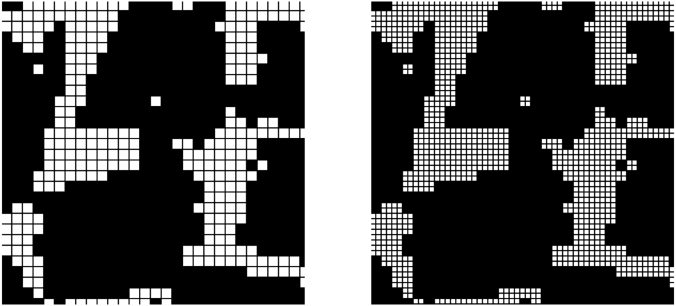


(b) Represented on 32^2

(c) Represented on 64^2

(d) Represented on 128^2

Part II: Numerical convergence

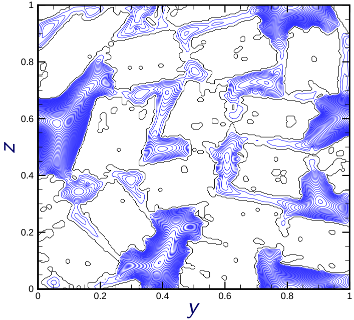


(a) Numerical resolution = geometry resolution

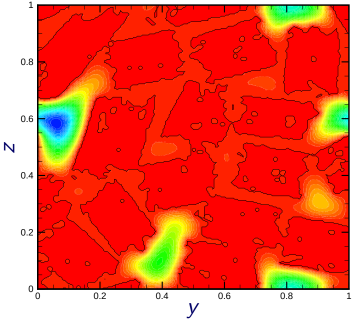
(b) Numerical resolution = 2 × geometry resolution

- Allows to study numerical convergence independently from geometrical convergence

Part II: Snapshot velocity & temperature field (256^2)

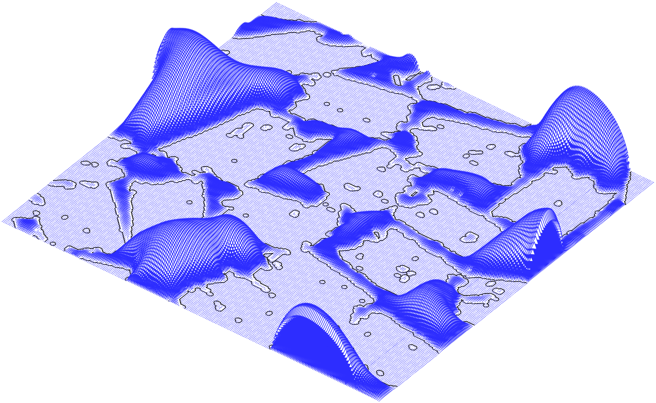


(a) Out-of-plane velocity contours



(b) Temperature (fluid & solid)

Part II: Snapshot velocity field (256^2)



Vector field out-of-plane velocity component

Part III: Conclusions & outlook

Conclusions:

- "Cartesian grid representation" suitable reconstruction of porous media described by imaging techniques
- Numerical approximation strategy reliably captures:
  - Laminar fluid dynamics using immersed boundary method
  - Conjugate heat transfer using single temperature equation

Outlook:

- Improve accuracy of immersed boundary method
- Incorporate chemical species interactions