Modified Rhie-Chow/PISO Algorithm for Collocated Variable Finite Volume Porous Media Flow Solvers

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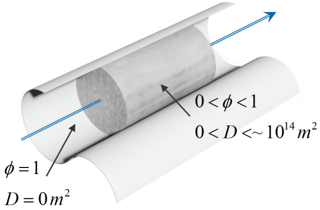
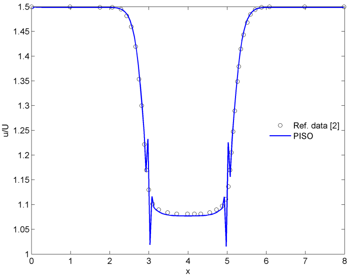
Outline

Background

- Flow simulations performed using standard segregated algorithms and interpolation schemes on collocated grids generates spurious oscillations in the velocity near porous interfaces





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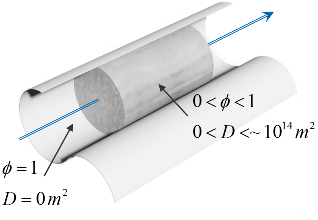
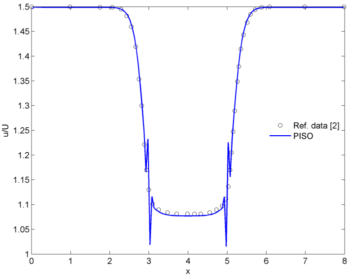
Background

- Flow simulations performed using standard segregated algorithms and interpolation schemes on collocated grids generates spurious oscillations in the velocity near porous interfaces
- Especially for high Reynolds number flows or for low permeability heterogeneous porous media





Objective

To develop a modified Rhie-Chow / PISO[1] segregated algorithm for low Mach number flow in heterogeneous, isotropic porous media

[1]PISO = Pressure-Implicit with Splitting of Operators

Objective

To develop a modified Rhie-Chow / PISO[1] segregated algorithm for low Mach number flow in heterogeneous, isotropic porous media, which by construction avoids the development of spurious oscillations

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[1]PISO = Pressure-Implicit with Splitting of Operators

Analysis of pressure-velocity coupling algorithm

to understand the origin of the pressure-velocity decoupling resulting in spurious oscillations

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Governing equations

Analysis of pressure-velocity decoupling

Volume averaged conservation equations for compressible flow and heat transfer in porous media under thermal non-equilibrium

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Volume averaged conservation equations for compressible flow and heat transfer in porous media under thermal non-equilibrium

- Mass conservation: $$\partial_t(\phi\langle\rho_\alpha\rangle^\alpha) + \partial_i(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle) = 0$$
- Momentum conservation: $$\partial_t(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle) + \partial_j(\phi^{-1}\langle\rho_\alpha\rangle^\alpha\langle u_j\rangle\langle u_i\rangle) = -\phi\partial_i\langle p_\alpha\rangle^\alpha + \partial_j\langle\tau_{ij}\rangle + \phi\langle f_i\rangle^\alpha - \phi D\langle u_i\rangle$$
- Fluid energy conservation: $$\langle c_{p,\alpha}\rangle^\alpha\left(\partial_t(\phi\langle\rho_\alpha\rangle^\alpha\langle T_\alpha\rangle^\alpha) + \partial_i(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle\langle T_\alpha\rangle^\alpha)\right) = \partial_i(\lambda_\alpha^e\partial_i\langle T_\alpha\rangle^\alpha) + h_{\alpha\beta}a_{\alpha\beta}(\langle T_\beta\rangle^\beta - \langle T_\alpha\rangle^\alpha) + \partial_t(\phi\langle p_\alpha\rangle^\alpha) + \langle u_i\rangle\partial_i\langle p_\alpha\rangle^\alpha$$
- Solid energy conservation: $$\langle c_{p,\beta}\rangle^\beta\langle\rho_\beta\rangle^\beta\partial_t((1-\phi)\langle T_\beta\rangle^\beta) = \partial_i(\lambda_\beta^e\partial_i\langle T_\beta\rangle^\beta) - h_{\alpha\beta}a_{\alpha\beta}(\langle T_\beta\rangle^\beta - \langle T_\alpha\rangle^\alpha)$$

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Overview of standard PISO algorithm

Analysis of pressure-velocity decoupling

Assume isothermal case for demonstration

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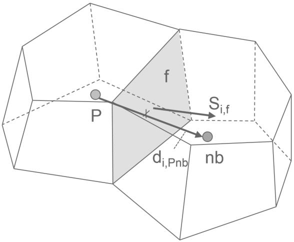
Assume isothermal case for demonstration

1. Predictor step:

   Solve discretized momentum equation

   $A_P u_{i,P} = H_{i,P} - \phi_P V_P^{-1} \sum_f p_f S_{i,f}$



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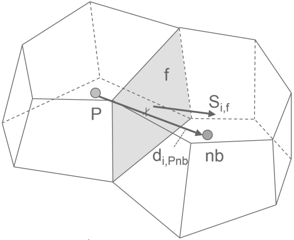

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2. Corrector steps:

   - **Solve discretized pressure equation**

     $V_P \partial_t(\phi_P \psi_P p_P) + \sum_f (\rho A^{-1} H_i)_f S_{i,f} = \sum_f (\rho A^{-1} \phi \partial_i p)_f S_{i,f}$

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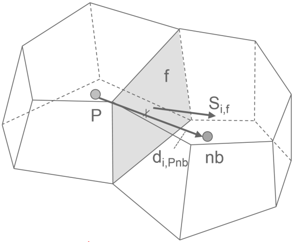

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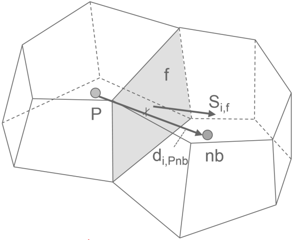

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   $A_P u_{i,P} = H_{i,P} - \phi_P V_P^{-1} \sum_f \color{red}{p_f} S_{i,f}$

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   - **Update mass flux at faces**

     $(\rho u_i)_f S_{i,f} = (\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1} \phi)_f (\partial_i p)_f S_{i,f}$

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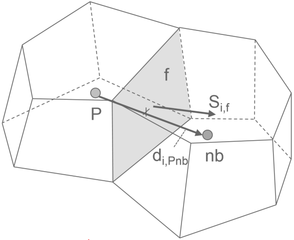

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   - **Correct velocity**

     $$u_{i,P} = A_P^{-1}(H_{i,P} - \phi_P V_P^{-1} \sum_f p_f S_{i,f})$$

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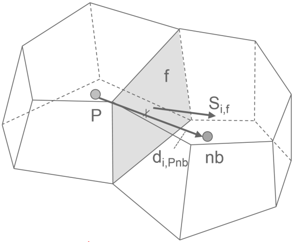

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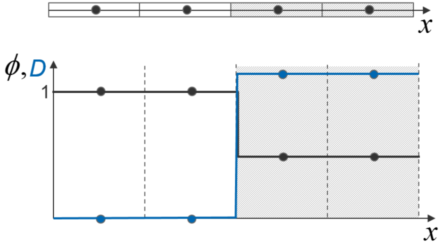
Interpolation from cell- to face-centers required

Pressure-velocity decoupling: 1D example

Analysis of pressure-velocity decoupling

Flow in a 1D heterogeneous porous medium:

- Discontinuous coefficients ϕ, D



Pressure-velocity decoupling: 1D example

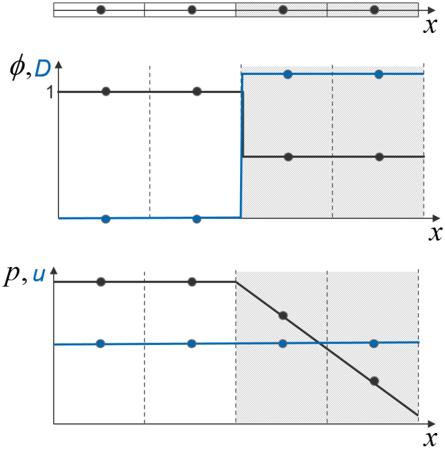
Analysis of pressure-velocity decoupling

Flow in a 1D heterogeneous porous medium:

- Discontinuous coefficients ϕ, D

Analytical solution:

- Constant velocity
- Linear pressure drop in porous medium



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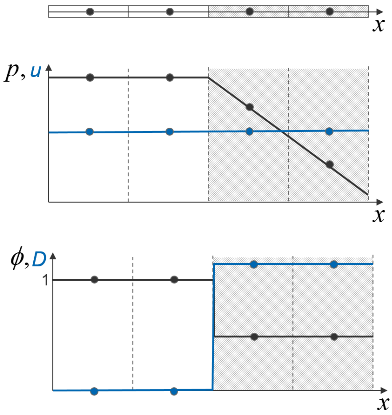
Pressure-velocity decoupling: 1D example

Analysis of pressure-velocity decoupling

Example of decoupling:

- Solving of momentum conservation equation in the predictor step:

$$A_P u_{i,P} = H_{i,P} - \phi_P V_P^{-1} \sum_f p_f S_{i,f}$$



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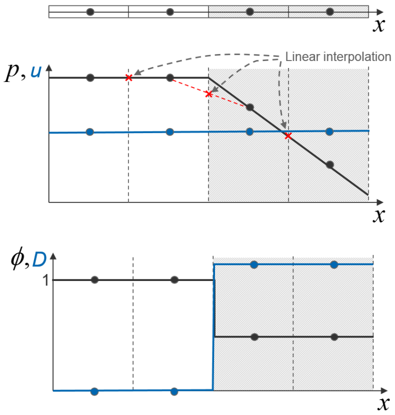

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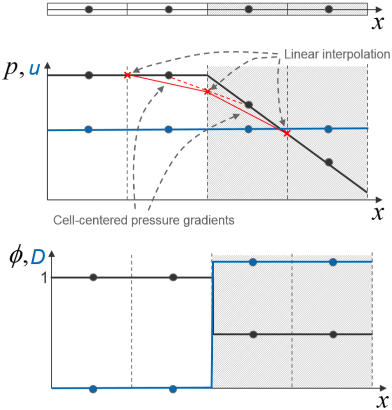

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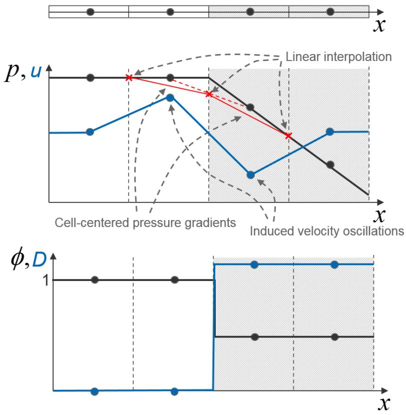
$$\Downarrow$$

Unphysical acceleration and deceleration in neighboring cells to the discontinuity



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Pressure-velocity decoupling: 1D example

Analysis of pressure-velocity decoupling

Other examples of decoupling due to inconsistent interpolation of discontinuous terms to the faces

- Pressure equation in the corrector step, since A also includes ϕ

  $V_P \partial_t(\phi_P \psi_P p_P) + \sum_f (\rho A^{-1} H_i)_f S_{i,f} = \sum_f (\rho A^{-1} \phi)_f (\partial_i p)_f S_{i,f}$
- Velocity correction:

  $u_{i,P} = A_P^{-1}(H_{i,P} - \phi_P V_P^{-1} \sum_f p_f S_{i,f})$

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Modified Rhie-Chow/PISO Algorithm for Collocated Variable Finite Volume Porous Media Flow Solvers

Redistributed Resistivity PISO (rdrPISO) algorithm

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Key elements of algorithm

Redistributed Resistivity PISO algorithm

Key elements:

1. Modified momentum / velocity corrections equation
2. Modified pressure equation (Rhie-Chow interpolation)

to ensure strong velocity-pressure coupling close to discontinuities

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Modified momentum / velocity correction equations

Redistributed Resistivity PISO algorithm

Cause for decoupling:

Cell-centered pressure gradients are not correctly balanced by the resistivity, in neighboring cells to the interface

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Modified momentum / velocity correction equations

Redistributed Resistivity PISO algorithm

Cause for decoupling:

Cell-centered pressure gradients are not correctly balanced by the resistivity, in neighboring cells to the interface

Remedy for decoupling:

- Reformulation of momentum equation to avoid:
  - explicit interpolation of porosity

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Modified momentum / velocity correction equations

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- Reformulation of momentum equation to avoid:
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PMI RESEARCH & DEVELOPMENT

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- Redistribution of resistivity to balance the cell-centered pressure gradients in neighboring cells to the discontinuity

PMI RESEARCH & DEVELOPMENT

Modified momentum / velocity correction equations

Redistributed Resistivity PISO algorithm

Reformulation of momentum equation

- Divide momentum equation by ϕ
- Move ϕ^{-1} out of convection term

so that no explicit interpolation of porosity is required for the discretization of the momentum equation

$$\frac{1}{\phi_P}\partial_t(\rho_P u_{i,P}) + \frac{1}{\phi_P^2 V_P}\sum_f (\rho u_j)_f u_{i,f} S_{j,f} = -(\partial_i p)_P + \frac{1}{\phi_P V_P}\sum_f \tau_{ij,f} S_{j,f} + f_{i,P} - D_P u_{i,P}$$

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- Keep resistivity term outside of A_P and $H_{i,P}$

$$(A_P + D_P)u_{i,P} = H_{i,P} - V_P^{-1}\sum_f p_f S_{i,f}$$

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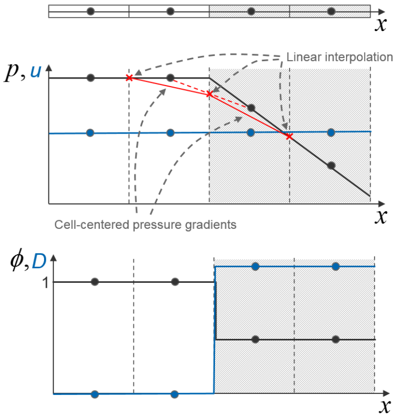
Modified momentum / velocity correction equations

Redistributed Resistivity PISO algorithm

Redistribution of resistivity

to balance the cell-centered pressure gradient calculated with central differences (same stencil)



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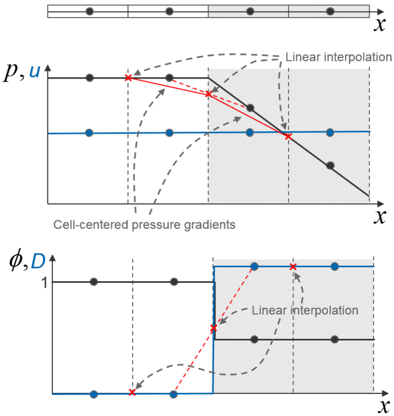

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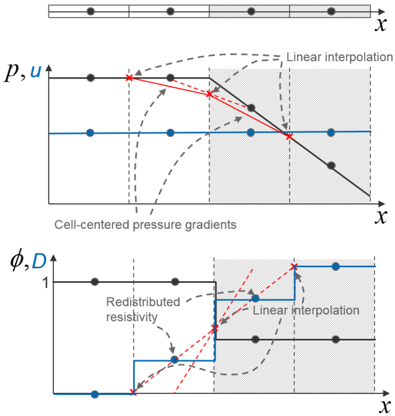

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Modified momentum / velocity correction equations

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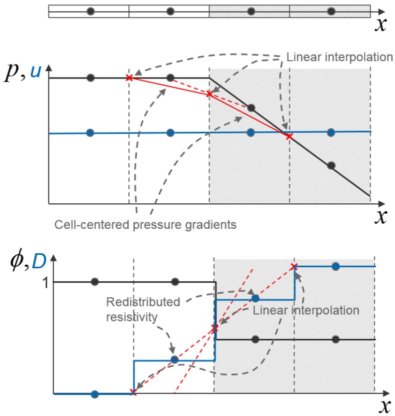
Modified momentum equation

$$(A_P + D_P^{rd}) u_{i,P} = H_{i,P} - V_P^{-1} \sum_f p_f S_{i,f}$$



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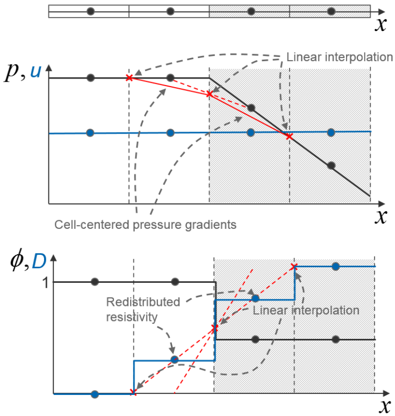

Modified momentum equation

$$(A_P + D_P^{rd}) u_{i,P} = H_{i,P} - V_P^{-1} \sum_f p_f S_{i,f}$$

Modified velocity correction equation

$$u_{i,P} = (A_P + D_P^{rd})^{-1} (H_{i,P} - V_P^{-1} \sum_f p_f S_{i,f})$$

PMI RESEARCH & DEVELOPMENT

Key elements of algorithm

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2. Modified pressure equation (Rhie-Chow interpolation)

to ensure strong velocity-pressure coupling close to discontinuities

PMI RESEARCH & DEVELOPMENT

Modified pressure equation

Redistributed Resistivity PISO algorithm

Cause for decoupling:

Inconsistent interpolation of discontinuous terms to the faces in the standard pressure equation, since A also includes ϕ

$$V_P \partial_t(\phi_P \psi_P p_P) + \sum_f (\rho A^{-1} H_i)_f \; S_{i,f} = \sum_f (\rho A^{-1} \phi)_f \; (\partial_i p)_f S_{i,f}$$

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Remedy for decoupling:

- Reformulation of **semi-discretized** momentum equation to avoid:
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- Reformulation of **semi-discretized** momentum equation to avoid:
  - explicit interpolation of porosity
  - having the resistivity as part of A and H_i

Modified pressure equation

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Remedy for decoupling:

- Reformulation of **semi-discretized** momentum equation to avoid:
  - explicit interpolation of porosity
  - having the resistivity as part of A and H_i
- Modification of Rhie-Chow interpolation to ensure consistent interpolation of discontinuous coefficients in pressure equation

Modified pressure equation

Redistributed Resistivity PISO algorithm

Reformulation of semi-discretized momentum equation

- Divide momentum equation by ϕ
- Move ϕ^{-1} out of convection term
- Keep resistivity term outside of A_P and $H_{i,P}$

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Modified pressure equation

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Reformulation of semi-discretized momentum equation

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$$u_{i,P} = B_P(A_P^{-1}H_{i,P} - A_P^{-1}(\partial_i p)_P)$$

where $B_P = (1 + D_P A_P^{-1})^{-1}$

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Modified pressure equation

Redistributed Resistivity PISO algorithm

Modification of Rhie-Chow interpolation

- Express the mass flux through a face as:

$$(\rho u_i)_f S_{i,f} = B_f[(\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1})_f (\partial_i p)_f S_{i,f}]$$

instead of: $(\rho u_i)_f S_{i,f} = (\rho B A^{-1} H_i)_f S_{i,f} - (\rho B A^{-1} \partial_i p)_f S_{i,f}$

where $B_f = (1 + D_f (A^{-1})_f)^{-1}$

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Modified pressure equation

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Modification of Rhie-Chow interpolation

- Express the mass flux through a face as:

$$(\rho u_i)_f S_{i,f} = B_f[(\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1})_f (\partial_i p)_f S_{i,f}]$$

instead of: $(\rho u_i)_f S_{i,f} = (\rho B A^{-1} H_i)_f S_{i,f} - (\rho B A^{-1} \partial_i p)_f S_{i,f}$

where $B_f = (1 + D_f (A^{-1})_f)^{-1}$

- Modified pressure equation:

$$V_P \partial_t (\phi_P \psi_P p_P) + \sum_f B_f (\rho A^{-1} H_i)_f S_{i,f} = \sum_f B_f (\rho A^{-1})_f (\partial_i p)_f S_{i,f}$$

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Summary of algorithm

Redistributed Resistivity PISO algorithm

Assume isothermal case for demonstration

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1. Update and redistribute resistivity:
   Compute D_P, D_f, D_P^{rd} and D_f^{rd}

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2. Predictor step:
   Solve modified discretized momentum equation
   $(A_P + D_P^{rd}) u_{i,P} = H_{i,P} - V_P^{-1} \sum_f p_f S_{i,f}$

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3. Corrector steps:
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   - Update mass flux at faces
     $(\rho u_i)_f S_{i,f} = B_f [(\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1})_f (\partial_i p)_f S_{i,f}]$

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   - Update mass flux at faces
     $(\rho u_i)_f S_{i,f} = B_f [(\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1})_f (\partial_i p)_f S_{i,f}]$
   - Correct velocity:
     $u_{i,P} = (A_P + D_P^{rd})^{-1} (H_{i,P} - V_P^{-1} \sum_f p_f S_{i,f})$

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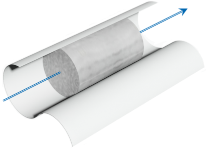
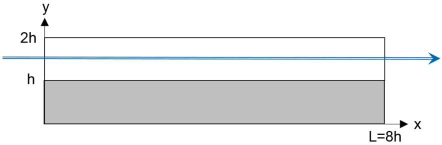
Validation of proposed algorithm

Porous plug case



Beaver-Joseph case



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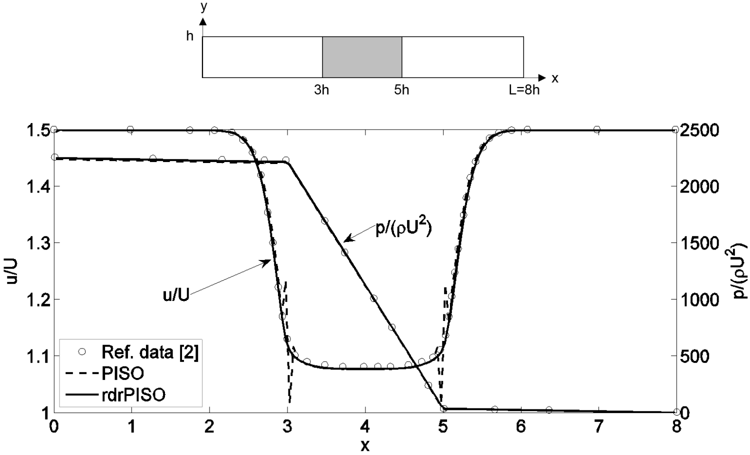

Centerline velocity for $Re = 1$ and $Da = 10^{-3}$

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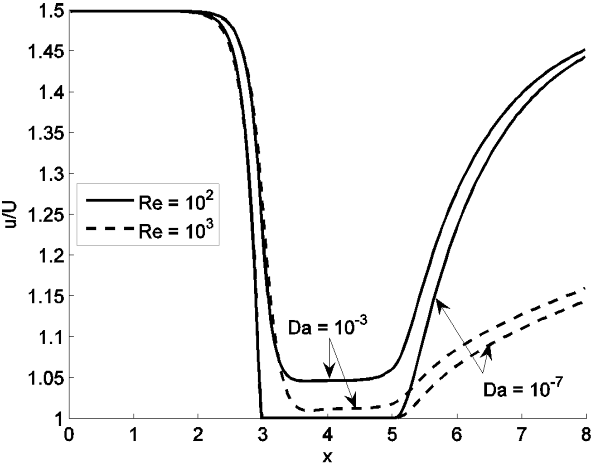
Porous plug case

Validation of proposed algorithm



Centerline velocity for $Re \in [10^2, 10^3]$, and $Da \in [10^{-3}, 10^{-7}]$

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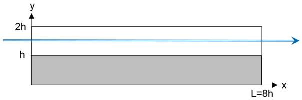
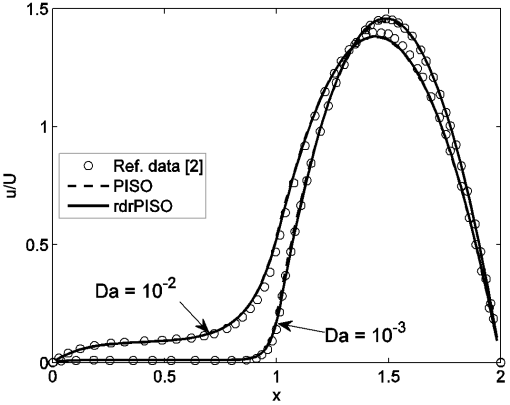
Beaver-Joseph case

Validation of modified algorithm





Velocity profiles for $Re = 1$
and $Da = 10^{-2}$ and $Da = 10^{-3}$

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Conclusions

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- Proposed a segregated algorithm for low Mach number flow and heat transfer in heterogeneous, isotropic porous media

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Conclusions

- Proposed a segregated algorithm for low Mach number flow and heat transfer in heterogeneous, isotropic porous media
- Successfully validated for incompressible and isothermal flow parallel and perpendicular to porous regions
- Robustness demonstrated for high Re flows up to $Re = 10^3$ and for Da numbers as low as $Da = 10^{-7}$

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Thank you for your attention!

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