Modeling Effective Properties of Porous Media Using Tomographic Reconstruction

David Lopez Penha\textsuperscript{a,}\textsuperscript{∗}, Bernard Geurts\textsuperscript{a}, Steffen Stolz\textsuperscript{a,}\textsuperscript{b}, Hans Kuerten\textsuperscript{a}
Markus Nordlund\textsuperscript{b}, Arkadiusz Kuczaj\textsuperscript{b}

\textsuperscript{a}Department of Applied Mathematics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands.
\textsuperscript{b}Philip Morris International R&D, Philip Morris Products S.A., Quai Jeanrenaud 5, 2000 Neuchâtel, Switzerland.

Background

- Porous media are encountered in a wide variety of technologies, e.g., for filtration, heat transfer, and chemical reaction.
- Large-scale computations are generally performed using transport equations in terms of mean flow variables.
- These equations contain unknown effective transport properties of the porous medium which are vital to performing accurate computations.

Objectives

- Develop a computer model for simulating the detailed transport of momentum and energy in a representative sample of the porous medium [1].
- Compute the directional permeability and Nusselt number as functions of the system properties.

Modeling strategy

- Use periodic unit cell that is representative of the porous medium under consideration.
- Solve the transport equations using periodicity conditions on \{u, p, T, T_s\}:
  \[
  \nabla \cdot u = 0, \quad \frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \frac{1}{Re} \nabla^2 u - \frac{1}{\epsilon} \Gamma(x), \quad \epsilon \ll 1
  \]
  \[
  \frac{\partial T}{\partial t} + \nabla \cdot (u T_s) = \frac{1}{Re Pr} \nabla^2 T_s, \quad \frac{\partial T_s}{\partial t} = \frac{1}{Re\lambda} \nabla^2 T_s + \frac{1}{\lambda} \nabla \cdot \mathbf{q}
  \]
  with \(\Gamma(x) = 1\) for \(x\) in the solid and \(\Gamma(x) = 0\) for \(x\) in the fluid. For the material properties: \(R_e \equiv \rho_s/\rho_f\) and \(R_s \equiv \lambda_s/\lambda_f\).
- Process the steady-state solution to obtain effective transport properties (direction \(n\)):
  \[
  \frac{1}{k_n} = - \frac{\langle n \cdot \nabla (p) \rangle}{|\langle u \rangle|}, \quad \text{Nu} = \frac{\int_{A_s} \nabla T_s \cdot dA}{\lambda_s (\langle T_s \rangle - \langle T \rangle)}
  \]

Structured models of porous media

Fig. 1. Fully-developed velocity vectors for an inline and a staggered arrangement of square rods (\(Re = 100\), \(\langle n \rangle = 1\)).

Fig. 2. Fields of fully-developed temperature fluctuation over a mean temperature field (\(Re Pr = 100; R_{\text{eq}} = R_s \rightarrow \infty\)).

Fig. 3. Permeability (\(\kappa\)) and Nusselt number (\(Nu\)) predictions along the \(x\)-axis versus the Reynolds number. \((\bigcirc)\)-marks represent the staggered arrangement and \((\times)\)-marks the inline arrangement (\(Pr = 1; R_{\text{eq}} = R_s \rightarrow \infty\)). Red data points represent inline data from Ref. [2].

Realistic 3D porous medium

- Geometry extracted from \(\mu\)CT-imaging (computer tomography).
- Represented on a Cartesian grid using the \(\Gamma\)-function.

Fig. 4. Raw and post-processed (black indicates solid objects; white are interstitial channels) \(\mu\)CT-image of a single cross section of a porous medium.

Fig. 5. Contour plot of the simulated out-of-plane velocity component and the solid-fluid temperature field.

Summary & outlook

- Developed computer model for momentum and energy transport.
- Improve accuracy of domain representation for application to realistic porous media.

References


\textsuperscript{∗}D.J. Lopez Penha (e-mail: d.j.lopezpenha@utwente.nl)